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Thermodynamic instability and critical fluctuations in dusty plasmas modelled as Yukawa OCP

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Abstract

The Helmholtz free energy of dusty plasmas is analysed as a one-component Yukawa system embedded in an ambient background plasma, and thermodynamic quantities are given in the form of simple interpolation formulae applicable in the domain of intermediate and strong coupling. By calculating the spectrum of the long wavelength fluctuations of the ambient plasma density, it is shown that there is a possibility of observing the divergence in the isothermal compressibility of the Yukawa OCP (a thermodynamic instability of a homogeneous phase) as an enhancement of fluctuation amplitudes and the critical condition is given as a combination of parameters for dusty plasmas.

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1. Introduction

The one-component plasma (OCP) is a system of charged particles of one species and a neutralizing background in which we pay attention mainly to the former. The dusty plasma is a charge-neutral mixture of micron-sized dust particles and an ambient plasma (of electrons and ions) and, in an approximation, dust particles in the dusty plasma may be regarded as a kind of OCP for which the ambient plasma serves as the background.

In dusty plasmas, dust particles usually have large negative charges and they are screened by ambient plasma particles. A simple approximation for the interaction between dust charges is the repulsive Yukawa potential for point particles and, within this approximation, we may regard the system of dust particles as a Yukawa OCP embedded in the background plasma of electrons and ions.

It is well known that there exist many effects in dusty plasmas which lead to effective forces on dust particles other than that simply expressed by the Yukawa OCP [1]. The ion flow induces an anisotropy, the inhomogeneity in the temperature leads to thermal forces, and the charge flow onto dust particles gives a kind of attraction. Our interest here, however, is in what happens when thermodynamic properties of the OCP are fully reflected in those of the dusty plasma as a whole and we adopt the simplest model, a Yukawa OCP embedded in a neutralizing ambient plasma. The result may serve as a reference in analysing real dusty plasmas.

In the classical (and also degenerate) OCP [2], the 'pressure' is negative for intermediate and strong coupling and, at some coupling, the inverse compressibility vanishes. The latter usually signals a thermodynamic instability of homogeneous phase and a phase separation.

The pressure is defined by the isothermal volume derivative of the Helmholtz free energy taken *maintaining the charge neutrality*. In the OCP, the background also changes its volume and the charge neutrality is satisfied when the volume of the system is changed [3]. However, since the background does not appear explicitly as independent degrees of freedom in the Hamiltonian, the contribution of the background is *not* included in the 'pressure' of the OCP. In real systems modelled as OCP, the background may be either the degenerate electrons for systems of nuclei, the trapping potential equivalent to the background charge for trapped ions, or the ion lattice for electron liquids. The background has its own pressure which is usually much larger than that of the OCP and masks the divergence of the OCP compressibility.

In this paper we point out a possibility of observing this divergence: when the coupling between dust particles is strong enough, their contribution to the pressure can overcome the masking effect of the ambient background plasma.

2. Adiabatic approximation

We consider a system composed of N_e electrons, N_i ions and N_d dust particles in a volume V. We denote their charges and number densities by $(-e, n_e = N_e/V)$, $(e, n_i = N_i/V)$ and $(-Qe, n_d = N_d/V)$, respectively, and assume the overall charge neutrality, $(-e)n_e + en_i + (-Qe)n_d = 0$. Taking the statistical average with respect to electrons and ions, we obtain the Helmholtz free energy of the system F as a sum of ideal gas contributions from electrons and ions $F_{id}^{(e)}(T_e, V, N_e) + F_{id}^{(i)}(T_i, V, N_i)$ and

$$-k_B T_d \ln\left[\frac{1}{(2\pi\hbar)^{3N_d} N_d!}\int\int_V \prod_i^{N_d} \mathrm{d}\mathbf{r}_i\, \mathrm{d}\mathbf{p}_i \exp[-(K_d + U_d)/k_B T_d]\right], \tag{1}$$

where K_d is the kinetic energy of dust particles and U_d is the Helmholtz free energy for a given configuration of dust particles. In our system, we may assign different temperatures $T_e \geqslant T_i \geqslant T_d$ to electrons, ions and dust particles, respectively, implicitly assuming slow energy relaxations between the three components. This assumption, however, is not essential to the conclusion. (If $T_e = T_i = T_d$, the critical condition may be reached even more easily.)

When we neglect the electron–electron, electron–ion and ion–ion correlation, we have [4, 5] $U_d = U_{\mathrm{coh}} + U_{\mathrm{sheath}}$. Here U_{coh} can be written in the form [6]

$$U_{\mathrm{coh}} = \frac{1}{2}\int\int_V \mathrm{d}\mathbf{r}\,\mathrm{d}\mathbf{r}'\,\frac{\exp(-|\mathbf{r}-\mathbf{r}'|/\lambda)}{|\mathbf{r}-\mathbf{r}'|}\rho(\mathbf{r})\rho(\mathbf{r}') - (\text{self-interaction}), \tag{2}$$

by the charge density composed of dust particles *and* the neutralizing background $\rho(\mathbf{r}) = \sum_{i=1}^{N_d}(-Qe)\delta(\mathbf{r}-\mathbf{r}_i) + Qen_d$ and λ is the screening length determined by electrons and ions [4, 5]. The term U_{sheath} is the free energy of the sheath around the dust particles and is independent of their configuration or radius in this approximation.

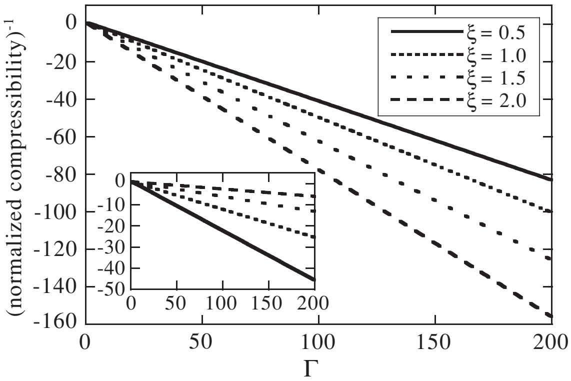

Figure 1. $-V(\partial p_d/\partial V)_T/n_d k_B T$ for the Yukawa OCP. The inset shows the values obtained without the contribution of U_{sheath}.

3. Thermodynamics of the strongly coupled Yukawa system

The three-dimensional Yukawa OCP is characterized by the parameters $\Gamma = (Qe)^2/ak_BT_d$ and $\xi = a/\lambda$, where $a = (3/4\pi n_d)^{1/3}$. When $\Gamma \gg 1$, the cohesive energy is well approximated by the Madelung energy of the Yukawa lattice. We find

$$\frac{U_{\text{coh}}}{N_d k_B T_d} \approx -0.896\Gamma \exp(-0.604\xi + 5.0 \times 10^{-4}\xi^4) + 0.54\Gamma^{1/4}\exp(-0.28\xi) \tag{3}$$

reproduces the cohesive energy [7, 4] within a relative error less than 1% for $10 \leqslant \Gamma$ and $0 \leqslant \xi \leqslant 5$. The coefficient -0.896 is chosen to fit both bcc and fcc values for $\xi = 0$.

Since $U_{\text{sheath}}/N_d k_B T_d = -(1/2)\Gamma\xi$, the nonideal part of the Helmholtz free energy of the system of dust particles is thus given by

$$\frac{F_d^{(d)} - F_{id}^{(d)}}{N_d k_B T_d} = f_d(\Gamma, \xi) = \int_0^\Gamma \frac{\mathrm{d}\Gamma}{\Gamma}\left(\frac{U_{\text{coh}}}{N_d k_B T_d} - \frac{1}{2}\Gamma\xi\right), \tag{4}$$

where $F_{id}^{(d)}$ is the ideal gas value. We will use the result for $10 \leqslant \Gamma$ and neglect the contribution from the domain of small Γ where the expression (3) does not apply. We also note that the effect of strong coupling on U_{sheath} is neglected within our model.

The inverse of the isothermal compressibility of the Yukawa OCP is shown in figure 1. We see that the inverse isothermal compressibility vanishes at small values of Γ and becomes negative.

4. Divergence of long-wavelength fluctuations in dusty plasmas

The Helmholtz free energy of dusty plasmas is given by the sum of contributions from electrons, ions and dust particles. The pressure of the system of charged particles p_{tot} is given by the sum of the contributions of the dust particles and those of ions and electrons, the latter being expressed by the ideal gas terms within our approximation. We thus have $p_{\text{tot}}/n_d k_B T_d = (n_e k_B T_e + n_i k_B T_i)/n_d k_B T_d + p_d/n_d k_B T_d$.

In the foregoing analyses, the density of the background is a constant Qen_d. We here consider the possibility of deformation of the ambient background plasma and derive the spectrum of fluctuations [8]. For this purpose, we represent the effect of the deformation in

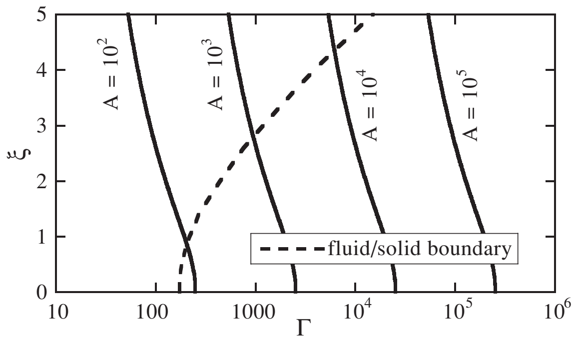

Figure 2. Conditions of the divergence of isothermal compressibility of dusty plasmas for different values of the ratio $A = (n_e k_B T_e + n_i k_B T_i)/n_d k_B T_d$.

terms of the background dust density $b(\mathbf{r})$ with charge $-(-Q)e = Qe$ introduced into the system.

The charge density is now written as $\rho(\mathbf{r}) + Qeb(\mathbf{r})$ which replaces $\rho(\mathbf{r})$ and $\rho(\mathbf{r}')$ on the right-hand side of (2). We calculate the Helmholtz free energy for the system described by this extended Hamiltonian regarding $b(\mathbf{r})$ as a perturbation. Expanding $b(\mathbf{r})$ into $b(\mathbf{r}) = (1/V)\sum_{\mathbf{k}} b(\mathbf{k})\exp(\mathrm{i}\mathbf{k}\cdot\mathbf{r})$, we have the long wavelength fluctuation spectrum of $b(\mathbf{k})$ similar to the critical opalescence [9] as

$$\frac{\langle |b(\mathbf{k})|^2\rangle}{N_d} \approx \frac{1 + \left(\frac{1}{k_D\lambda}\right)^2 \left(\frac{\partial p_{\text{tot}}}{\partial n_d}\right)_T \frac{1}{k_B T} + \mathcal{O}(k^2)}{\left(\frac{\partial p_{\text{tot}}}{\partial n_d}\right)_T \frac{1}{k_B T} + \mathcal{O}(k^2)}. \tag{5}$$

When $(\partial p_{\text{tot}}/\partial n_d)_T \to 0$, the long wavelength fluctuations are enhanced and eventually diverge in proportion to the compressibility.

In figure 2, we plot the condition for the divergence of the compressibility. The condition depends on the parameter $A = (n_e k_B T_e + n_i k_B T_i)/n_d k_B T_d$ which is usually much larger than unity: the number densities as well as the temperature of electrons and ions are larger than those of dust particles. On the other hand, the coupling parameter Γ can take very large values due to large negative charges on dust particles. We may therefore expect that the above condition might be met under appropriate experimental conditions. The condition for the liquid–solid transition of the Yukawa system is also shown in figure 2. We observe that dust particles can either be solid or liquid depending on experimental parameters when this condition is satisfied.

5. Conclusions

Enhanced fluctuations associated with a thermodynamic instability in dusty plasmas are shown. Such a possibility in the classical OCP was pointed out some time ago by introducing a freely deformable imaginative background [8]. We have here derived the conditions for dusty plasmas modelled as the Yukawa OCP embedded in a background.

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